

Name: Ameera Ahmed Al Ameer

Title: EXTENSION OF PLURISUBHARMONIC CURRENTS
ACROSS THE ZERO SET OF A k -CONVEX FUNCTION

Major: Mathematics

Date: June 2009

Supervisor: Dr. Hassine El Mir
Professor in Pure Mathematics

Abstract:

This thesis is concerned with plurisubharmonic currents. The main originality in this work is the following theorem:

Let $\Omega \subset \mathbb{C}^n$ be a domain, for each $z = (x_1 + iy_1, \dots, x_n + iy_n) \in \Omega$ consider $u(z) = y_1^2 + y_2^2 + \dots + y_n^2$ and $A = \Omega \cap \mathbb{R}^n$ i.e.

$$A = \{(x_1 + iy_1, \dots, x_n + iy_n) \in \Omega : y_1^2 + y_2^2 + \dots + y_n^2 = 0\}$$

Let S be a positive current on Ω and let T be a positive current of bidimension (p, p) on $\Omega \setminus A$. Then

1. If $p \geq 1$ and $dd^c T \leq S$, the trivial extension \tilde{T} by zero across A exists.
 2. If $p \geq 2$ and $dd^c T \geq -S$:
- (i) There exists a smooth sequence (ρ_n) which vanishes in a neighborhood of A and converges uniformly on the compact subsets of $\Omega \setminus A$ to $1_{\Omega \setminus A}$ such that

$$d\tilde{T} = \lim_{n \rightarrow \infty} \rho_n dT,$$

- (ii) The trivial extension $\widehat{dd^c T}$ exists. And we have $\widehat{dd^c T} = dd^c \tilde{T}$.

This theorem generalizes some recent results. In particular, if T is a positive closed, then \tilde{T} is also a closed positive current. This result is done by El Mir in 1982.

The particular case, when $dd^c T \leq 0$ or $dd^c T \geq 0$, is done by Sibony in 1985, but with the additional technical hypothesis that dT is of order zero.

In 2003, EL Mir, Dabbek and Elkhadhra proved the theorem with the hypothesis that $dd^c T \leq 0$ or $\widehat{dd^c T}$ exists, but without requiring anything from dT .